



CONTROL DESIGN FOR GENERALIZED NORMAL MODE OPERATION OF BIAS MOMENTUM SATELLITES

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(Received September 1995; in final form June 1996)

Abstract: An attitude controller for a 3-axis stabilized, Earth-oriented bias momentum spacecraft is described, where only 2-axis attitude measurements from the Earth sensor are available. In contrast to classical spacecraft, that are controlled with respect to a fixed orbital Earth pointing reference frame, (possibly large-angle) time-varying reference signals are considered here, i.e. the control task consists of a tracking problem. The controller design consists of a decoupling controller and axis-related PID controllers based on yaw observer estimates.

Keywords: Earth pointing, yaw observer, roll-tracking, time-varying reference

NOTATION

\underline{a}	vector \underline{a}
\underline{a}^T	transposed vector \underline{a}
$\dot{\underline{a}}$	derivative of \underline{a} with respect to time
$\underline{\tilde{a}}$	cross-product matrix of \underline{a}
\underline{a}_M	measurement of \underline{a}
A	matrix A
T_a^b	transformation matrix from system a to b
E	identity matrix
$\underline{\omega}^a$	rate, expressed in system a
$\underline{\omega}$	rate without superscript: expressed in body system
c, s	cosine, sine
LEO	low Earth orbit
GEO	geostationary orbit

the orbital plane, which leads to observability of the yaw motion by the roll measurement. An early publication in this field is (Dougherty, *et al.* 1968), which is well-known as the "Whecon"-principle.

This paper describes a control design approach for a generalized Earth-pointing control mode with 2-axis Earth sensor measurements only and bias momentum coupling, where time-varying attitude reference signals with respect to an Earth-pointing coordinate system are considered. This means that the control task here tackles a tracking problem in addition to a disturbance rejection problem. A possible control task is shown in Fig. 1. The desired spacecraft attitude, here a roll-bias angle α and zero pitch and yaw angles, can be expressed as a time-varying reference attitude with respect to the orbital Earth-pointing coordinate system (x_o, y_o, z_o) . Roll-tracking is necessary in the case of inclined orbit operations of geosynchronous satellites, where proper Earth-orientation has to be maintained, for antenna pointing purposes.

Another example includes small satellites in low Earth orbits, that use - besides the solar array rotation - one degree of freedom around the satellite yaw axis for the optimal orientation of the solar panels, i.e. to ensure that the sun vector is always (nearly) perpendicular to the panel surface.

1. INTRODUCTION

Earth-oriented, three-axis stabilized satellites generally have no continuous yaw attitude information available, or even no yaw measurement at all. This is especially true for commercial communication satellites, which have to be designed under stringent economic conditions. The common approach to achieve 3-axis stabilization with a 2-axis attitude sensor only (Earth sensor) is to establish a bias momentum perpendicular to

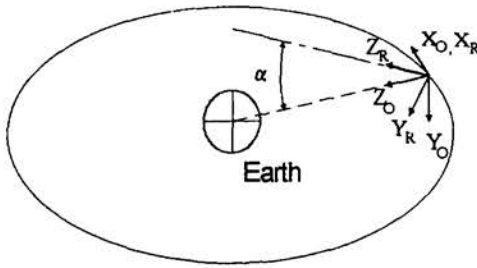


Figure 1: Orbit and reference coordinate systems.

The subsequent explanations cover, as far as possible, a general case. Examples are given for the abovementioned application of roll-tracking operations.

The minimum sensor and actuator hardware configuration which is necessary for the realization of this attitude-control approach consists of the following components:

- a) A set of wheels that span the 3-dimensional space, i.e., in practice linear actuators which produce torques around the 3 spacecraft axes. Usually 4 wheels are used for redundancy.
- b) An Earth sensor that delivers 2-axis attitude information around the roll- and pitch axes.

Additionally, an actual spacecraft has to be equipped with actuators for angular momentum control, such as magnetic torquers, thrusters, and/or solar array drives for solar torque compensation, depending on the spacecraft mission. The remainder of this paper deals with the attitude control.

2. SPACECRAFT DYNAMICS AND KINEMATICS

In this section the system equations describing a spacecraft's dynamics and kinematics are presented.

2.1 Transformations between the coordinate systems involved

Transformation from orbit system to reference system. The spacecraft is rotated from the orbit system by time-varying bias Euler angles $\gamma(t)$ about the yaw-axis, $\beta(t)$ about the pitch-axis and $\alpha(t)$ about the roll-axis, with corresponding transformation matrices $T_\alpha, T_\beta, T_\gamma$:

$$T_\alpha = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha & s\alpha \\ 0 & -s\alpha & c\alpha \end{bmatrix}; \quad T_\beta = \begin{bmatrix} c\beta & 0 & -s\beta \\ 0 & 1 & 0 \\ s\beta & 0 & c\beta \end{bmatrix} \quad (1a;b)$$

$$T_\gamma = \begin{bmatrix} c\gamma & s\gamma & 0 \\ -s\gamma & c\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1c)$$

Applying the rotation in this order yields the transformation matrix from orbit to reference system

$$T_O^R = T_\alpha T_\beta T_\gamma = \begin{bmatrix} c\beta c\gamma & c\beta s\gamma & -s\beta \\ s\alpha s\beta c\gamma - c\alpha s\gamma & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha c\beta \\ c\alpha s\beta c\gamma + s\alpha s\gamma & c\alpha s\gamma s\beta - s\alpha c\gamma & c\alpha c\beta \end{bmatrix} \quad (2)$$

with column vectors

$$T_O^R = [t_1 \ t_2 \ t_3] \quad (3)$$

Transformation from reference system to body system.

The satellite deviates from its reference attitude by the Euler angles $\varphi = [\phi \ \Theta \ \psi]^T$, which the controller tries to suppress in the presence of disturbances. For small Euler angles the transformation matrix from the reference system to the body system can be linearized to give

$$T_R^B = E - \tilde{\varphi} \quad (4)$$

2.2 Kinematics

The dynamic behaviour of the spacecraft has to be described in terms of Euler angles. Therefore, the body's angular velocity and the body's angular acceleration appearing in the angular momentum equation need to be expressed by the Euler angles.

The absolute body angular velocity of the spacecraft $\underline{\omega}$, expressed in the body system, can be split into three parts

$$\underline{\omega} = \underline{\omega}_O + \underline{\omega}_R + \dot{\varphi} \quad (5)$$

where $\underline{\omega}_O$ is the orbit angular velocity of the orbit system relative to the inertial system, $\underline{\omega}_R$ is the reference angular velocity of the reference system relative to the orbit system, and $\dot{\varphi}$ describes the body angular velocity relative to the reference system.

Orbit angular velocity $\underline{\omega}_O$ The orbit angular velocity expressed in the orbit system is $\underline{\omega}_O^O = [0 \ -\omega_0 \ 0]^T$, and can be expressed in the body system by applying two subsequent transformations:

$$\underline{\omega}_O = T_R^B T_O^R \underline{\omega}_O^O = -[E - \tilde{\varphi}] t_2 \omega_0 \quad (6)$$

Reference angular velocity $\underline{\omega}_R$ Defining the vectors

$$\alpha = \begin{pmatrix} \alpha \\ 0 \\ 0 \end{pmatrix}; \quad \beta = \begin{pmatrix} 0 \\ \beta \\ 0 \end{pmatrix}; \quad \gamma = \begin{pmatrix} 0 \\ 0 \\ \gamma \end{pmatrix}, \quad (7a;b;c)$$

$\underline{\omega}_R$ can be written as

$$\underline{\omega}_R = T_R^B \underline{\omega}_R^R = [E - \tilde{\varphi}] \underline{\omega}_R^R \quad (8)$$

with

$$\underline{\omega}_R^R = \dot{\alpha} + T_\alpha \dot{\beta} + T_\beta \dot{\gamma}. \quad (9)$$

2.3 Expressing the Euler equation in terms of measured roll and pitch angles

The well-known Euler equation describing the behaviour of a rigid body tumbling alone in space (Wittenburg, 1977) is

$$I \dot{\underline{\omega}} + \tilde{\omega} I \underline{\omega} + \tilde{\omega} \underline{h} = -\dot{\underline{h}} + \underline{T}_C + \underline{T}_D, \quad (10)$$

where the variables in eq. (9) have the following meaning:

- I - inertia matrix with respect to principal axes
- \underline{h} - angular momentum of wheels
- \underline{T}_C - external control torque
- \underline{T}_D - external disturbance torque.

In the next section a control law is developed which linearizes the Euler equation. However, it is assumed, that only roll and pitch angles are known by measurement. Because these signals will be used for the control law, the Euler equation (10) is rewritten in terms of the new measurement vector $\underline{\varphi}_M = (\phi \ \theta)^T$.

Therefore, the spacecraft angular velocity vector $\underline{\omega}$ in eq. (5) is only partially known, and is replaced by $\underline{\omega}_M$:

$$\underline{\omega}_M = \underline{\omega}_{OM} + \underline{\omega}_{RM} + \dot{\underline{\varphi}}_M. \quad (11)$$

The measured part of the orbit angular velocity $\underline{\omega}_{OM}$ is obtained by replacing $\underline{\varphi}$ in eq. (6) by $\underline{\varphi}_M$:

$$\underline{\omega}_{OM} = -[E - \tilde{\varphi}_M] \underline{\omega}_0. \quad (12)$$

Defining the vector $\underline{\Psi} = [0 \ 0 \ \psi]^T$, and adding

$$\Delta \underline{\omega}_0 = \underline{\Psi} \underline{\omega}_0 \quad (13)$$

recovers $\underline{\omega}_0$. Similarly, $\underline{\omega}_{RM}$ is obtained by replacing $\underline{\varphi}$ in eq. (8) by $\underline{\varphi}_M$,

$$\underline{\omega}_{RM} = [E - \tilde{\varphi}_M] \underline{\omega}_R^R, \quad (14)$$

and adding

$$\Delta \underline{\omega}_R = -\underline{\Psi} \underline{\omega}_R^R \quad (15)$$

recovers $\underline{\omega}_R$.

The spacecraft angular velocity vector $\underline{\omega}$ can be written as

$$\underline{\omega} = \underline{\omega}_M + \Delta \underline{\omega} \quad (16)$$

with

$$\Delta \underline{\omega} = \dot{\underline{\Psi}} + \Delta \underline{\omega}_0 + \Delta \underline{\omega}_R, \quad (17)$$

yielding the spacecraft angular acceleration

$$\dot{\underline{\omega}} = \dot{\underline{\omega}}_M + \Delta \dot{\underline{\omega}}. \quad (18)$$

The measured spacecraft angular acceleration $\dot{\underline{\omega}}_M$ is further split into

$$\dot{\underline{\omega}}_M = \dot{\underline{\omega}}_{CM} + \dot{\underline{\varphi}}_M \quad (19)$$

with

$$\dot{\underline{\omega}}_{CM} = \dot{\underline{\omega}}_{OM} + \dot{\underline{\omega}}_{RM}, \quad (20)$$

because the signal $\dot{\underline{\omega}}_{CM}$ (rather than $\dot{\underline{\omega}}_M$) is decoupled in order to leave second derivatives of ϕ and θ in the system equation.

Before eqs (16-18) and (19) are inserted into Euler equation (10), the second term in eq. (10) is simplified: Inserting eq. (16) in this term gives

$$\begin{aligned} \tilde{\omega} I \underline{\omega} &= [\tilde{\omega}_M + \Delta \tilde{\omega}_M] I (\underline{\omega}_M + \Delta \underline{\omega}_M) \\ &\approx \tilde{\omega}_M I \underline{\omega}_M + [\tilde{\omega}_M I - \tilde{I} \tilde{\omega}_M] \Delta \underline{\omega}. \end{aligned} \quad (21)$$

Inserting eqs (16), (18), (19) and (21) in eq. (10) gives, after rearranging terms,

$$\begin{aligned} I \dot{\underline{\omega}}_{CM} + \tilde{\omega}_M I \underline{\omega}_M + \tilde{\omega}_M \underline{h} + \\ I \dot{\underline{\varphi}}_M + I \Delta \dot{\underline{\omega}} + [\tilde{\omega}_M I - \tilde{I} \tilde{\omega}_M - \tilde{h}] \Delta \underline{\omega} = \\ -\dot{\underline{h}} + \underline{T}_C + \underline{T}_D. \end{aligned} \quad (22)$$

3. DECOUPLING AND TRACKING CONTROL

In this section a wheel control torque is developed to satisfy two design objects: first, to decouple the yaw dynamics from the roll and pitch dynamics, and second, to enable tracking of arbitrary bias angles $\alpha(t)$, $\beta(t)$ and $\gamma(t)$. The wheel torque is decomposed into

$$-\dot{\underline{h}} = -\dot{\underline{h}}_D - \dot{\underline{h}}_C \quad (23)$$

where $\dot{\underline{h}}_D$ is the decoupling and $\dot{\underline{h}}_C$ is the tracking control part.

3.1 Decoupling control law

In the reformulated Euler equation (22) the measurable terms except for the term "I $\ddot{\omega}_M$ " are put into $-\dot{h}_D$, i.e.

$$-\dot{h}_D = I\dot{\omega}_{CM} + \tilde{\omega}_M J \omega_M + \tilde{\omega}_M \dot{h} \tag{24}$$

Inserting eq. (24) in eq. (22) results in the decoupled system equation

$$I\ddot{\omega} + [\tilde{\omega}_M J - \tilde{I}\tilde{\omega}_M - \tilde{h} - \tilde{I}\tilde{\omega}_0 + I\tilde{\omega}_R^R] \dot{\psi} + [\tilde{\omega}_M J - \tilde{I}\tilde{\omega}_M - \tilde{h}] [-\tilde{I}\tilde{\omega}_0 + \tilde{\omega}_R^R] \dot{\psi} - \dot{h}_c + T_c + T_D = \tag{25}$$

The matrices in brackets in eq. (25) still contain Euler angles ϕ and θ . Here, they can be ignored, because they are subsequently multiplied with ψ and $\dot{\psi}$, respectively, resulting in "small" products which can be neglected. There remains only a one-way-directed coupling between yaw dynamics and roll/pitch dynamics: yaw couples in roll/pitch, but not vice versa.

3.2 Tracking control and yaw estimation

Equation (25), which describes the already partially decoupled plant dynamics with respect to the reference attitude, will now be divided into two subsystems according to the roll/pitch motion and the yaw motion. Remembering that the first two components of ψ are zero, eq. (25) can be formally rewritten as

$$\begin{pmatrix} \ddot{\phi} \\ \ddot{\theta} \end{pmatrix} + c_1(t)\dot{\psi} + d_1(t)\psi = -\dot{h}_{c1}^* + T_{c1}^* + T_{D1}^* \tag{26}$$

$$\ddot{\psi} + c_2(t)\dot{\psi} + d_2(t)\psi = -\dot{h}_{c2}^* + T_{c2}^* + T_{D2}^* \tag{27}$$

where $c_1(t)$, $d_1(t)$ are (2x1) vectors, $c_2(t)$, $d_2(t)$ are scalars; the "*" - superscript indicates that the torques on the right-hand sides of eqs (26) and (27) are normalized with respect to the diagonal elements of I. For a properly established bias momentum along the orbit normal vector, evaluation of these coefficients shows the following properties:

- (i) $d_2(t)$ and $|c_1(t)|$ have a dominating bias. This means that at least one component of $c_1(t)$ is relatively large. Comparison: for an Earth pointing geostationary satellite, $c_1 = (h_y, 0)^T$ and $d_2 = \omega_0 h_y$, see (Dougherty, *et al.*, 1968).
- (ii) $|d_1(t)|$ is relatively small. For the geostationary case, $d_1 = (0, 0)^T$.
- (iii) $c_2(t) = 0$. This reflects the fact that a bias momentum satellite is a gyroscopic system, i.e. there is no dissipative damping.

Eqs (26) and (27) demonstrate that the yaw dynamic couples in the roll/pitch dynamics, but not vice versa.

Axis-related controllers. The roll/pitch subsystem is controlled by a standard PID control law

$$-\dot{h}_{c1} = -K_p \begin{pmatrix} \phi \\ \theta \end{pmatrix} - K_D \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \end{pmatrix} - K_I \int \begin{pmatrix} \phi \\ \theta \end{pmatrix} dt \tag{28}$$

using the measurements of the Earth sensor. Roll and pitch angle derivatives are obtained by numerical differentiation (filtering). K_p , K_D and K_I are diagonal (2x2) gain matrices; remember that ϕ and θ are the deviations from the time-varying reference attitude system.

The yaw axis is controlled by a PD control law

$$-\dot{h}_{c2} = -k_p \psi - k_D \dot{\psi} \tag{29}$$

where k_D , k_p are scalar gains and ψ , $\dot{\psi}$ are estimates of the yaw state with respect to the reference yaw attitude. They are provided by the yaw observer which is discussed below.

Yaw observer. Basically, the yaw state estimates are obtained by a reduced-order observer. Although there are standard design procedures - at least for time-invariant systems - the subsequent explanations are based on an "engineering approach" rather than a strict mathematical approach, because it considers a feel for the physics of bias momentum satellites and it is very similar for both cases, i.e., time-varying and time-invariant reference attitudes.

For simplicity, T_c , T_D and K_I are assumed to be zero for the time being. With properties (i), (ii), and eq. (28), eq. (26) can be rewritten as

$$c_1(t) \dot{\psi} = - \begin{pmatrix} \ddot{\phi} \\ \ddot{\theta} \end{pmatrix} + K_D \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \end{pmatrix} + K_p \begin{pmatrix} \phi \\ \theta \end{pmatrix} \tag{30}$$

Then Laplace transformation of eq. (30) for a particular time instant t_0 yields

$$\begin{pmatrix} \phi \\ \theta \end{pmatrix} = - [Es^2 + K_D s + K_p]^{-1} c_1(t_0) \dot{\psi} \tag{31}$$

i.e. $(\phi \ \theta)^T$ can be regarded as a (delayed) measurement of $\dot{\psi}$, scaled by $c_1(t)$. In order to obtain a pseudo-measurement $\dot{\psi}_m$, eq. (31) is multiplied by a vector $\underline{w} = (w_1 \ w_2)^T$ leading to

$$\dot{\psi}_m = \underline{w}^T \begin{pmatrix} \phi \\ \theta \end{pmatrix} = - \underline{w}^T(t_0) [Es^2 + K_D s + K_p]^{-1} c_1(t_0) \dot{\psi} \tag{32}$$

The vector \underline{w} can be chosen in such a way, that the steady-state transfer function in eq. (32) from $\dot{\psi}$ to $\dot{\psi}_m$ equals one, i.e.

$$\underline{w} = -\frac{K_p^{-1} \underline{\epsilon}_1}{|K_p^{-1} \underline{\epsilon}_1|^2} \quad (33)$$

Because of property (i) and the diagonality-property of K_p , the numerator in eq. (33) is nonzero, and in all cases a vector \underline{w} can be computed according to eq. (33).

Another possibility to generate $\underline{\psi}_m$ is to switch between the first and second rows of eqs (31) and (32), respectively, depending on the components of $\underline{\epsilon}_1$. In this case the corresponding components of \underline{w} have to be zero.

If the second derivative of $(\phi \ \theta)^T$ in eq. (30) is ignored, then even an unfiltered pseudo-measurement $\underline{\psi}_m$ is available, because the roll and pitch angles and their derivatives are known. Experience shows that this is possible for many applications.

Now $\underline{\psi}_m$ can be used to design a standard observer for the yaw angle ψ , based on the plant model, eq. (27), which is decoupled from the roll/pitch subsystem. Observability can be verified using property (i).

In case of nonzero external torques $\underline{T}_C, \underline{T}_D$, their known contributions can be considered in eq. (32), their unknown contributions result in errors of the pseudo-measurement $\underline{\psi}_m$. For nonzero K_i , the integrals of the roll and pitch angles have to be used for measurement purposes, instead of the attitude angles.

Stability analysis. Due to the periodic variation of the parameters of the plant dynamics (26) and (27), the closed-loop stability analysis can be performed by means of Floquet theory, see e.g. (Vidyasagar, 1993).

4. SIMULATION RESULTS

The simulation assumptions and results are taken from (Surauer, 1995).

4.1 LEO-application

Some typical applications for LEO-control tasks are Earth pointing (i.e., pitch tracking), and roll and yaw tracking. For the simulations, the corresponding spacecraft parameters, disturbance torques and orbit parameters are listed below:

Inertia matrix $I = \text{diag} \{850, 200, 860\} \text{ Nms}^2$.
 Orbit frequency $\omega_0 = 2\pi / (7200 \text{ s})$.
 Disturbance torque $\underline{T}_D = \underline{a}_0 + \underline{a}_1 \cos \omega_0 t + \underline{a}_2 \sin \omega_0 t$
 with $\underline{a}_0^T = [4 \ 20 \ 10] 10^{-6} \text{ Nm}$,
 $\underline{a}_1^T = [-15 \ 10 \ 0] 10^{-6} \text{ Nm}$,
 $\underline{a}_2^T = [0 \ 10 \ 15] 10^{-6} \text{ Nm}$.

Bias angular momentum $h_y^0 = -12 \text{ Nms}$.
 IRES noise: $3\sigma = 0.1 \text{ degree}$.

Angular momentum control is performed by taking

magnetic torquers to generate external control torques. An example of a roll-tracking maneuver is shown in Figs 2 - 4.

Fig. 2 shows the roll-reference α and the controlled roll attitude $\alpha + \phi$. With this ordinate scaling no difference between the two signals can be noticed.

In Fig. 3 the time history of attitude control errors in roll, pitch and yaw is plotted.

Fig. 4 shows the wheel angular momentum during the roll-tracking maneuver.

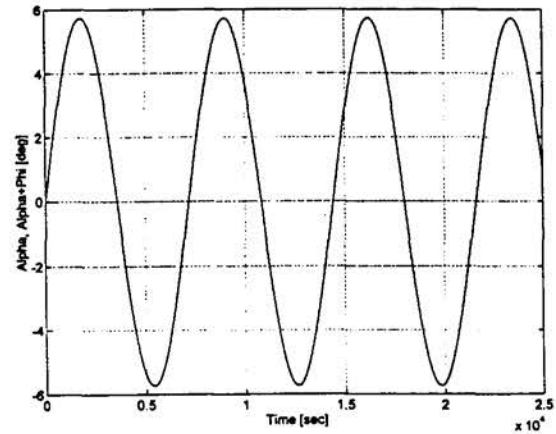


Figure 2: Roll reference and controlled roll attitude.

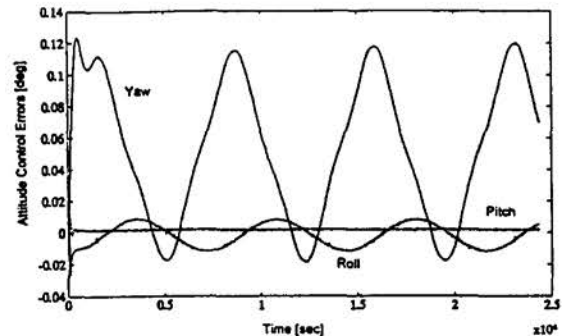


Figure 3: Roll, pitch and yaw errors.

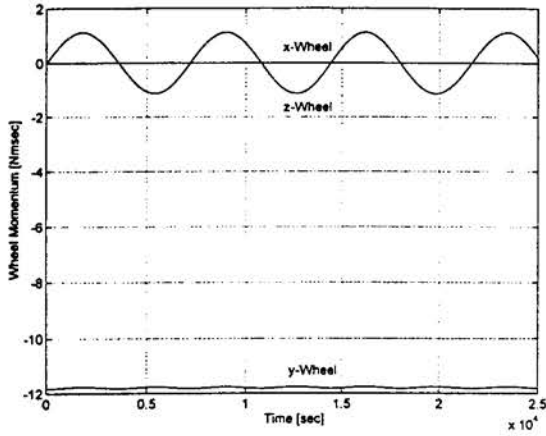


Figure 4: Resulting angular momentum of wheels.

4.2 GEO application

In order to demonstrate the benefits of the yaw observer derived in Section 3.2, the transient behaviour of a GEO satellite with large initial yaw angle ($\psi_0 = 25$ degree) is investigated. No disturbance torque is assumed. The orbit frequency is now $\omega_0 = 2\pi / (24 \text{ h})$; the remaining data are the same as in Section 4.1. The time history of ψ is shown in Fig. 5.

Steady-state conditions are reached after 2 hours, whereas a "whecon" controller needs 6 hours to reach steady state.

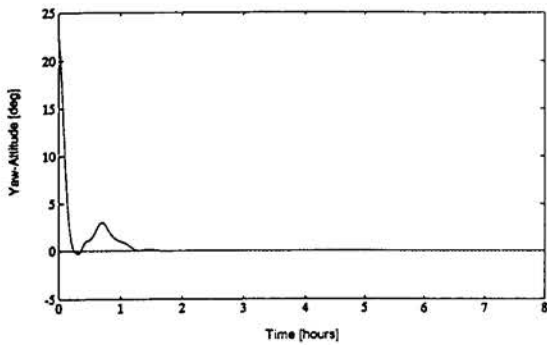


Figure 5: Yaw-transient behaviour of a GEO satellite.

5. CONCLUSIONS

A control law for Earth-oriented momentum bias satellites with time-varying attitude reference signals was derived. Such reference signals have to be applied, for example, for roll-tracking maneuvers. The corresponding minimum hardware configuration consists basically of a 2-axis Earth sensor, and wheels that provide a linear torque capability around all 3 spacecraft axes.

The control laws consist of a nonlinear decoupling part that leads to linear, but time-varying, plant dynamics, and axis-related PID controllers for the control with respect to the reference attitude. The yaw state estimates are provided by an observer for the time-varying plant.

This approach has the advantage that it can be applied to a more general class of normal mode operations. It also results in an improved transient behaviour for nonzero initial conditions, without serious degradation of the disturbance-rejection properties. This was demonstrated by simulation time histories for a classical geostationary satellite in an equatorial orbit, revealing fast transient behaviour, and for a satellite in LEO performing roll-tracking maneuvers.

ACKNOWLEDGEMENT

The authors wish to thank Mr. Burazanis for his help in dealing with the wordprocessing and the drawing software.

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