Student Reasoning in Fluid Dynamics: Bernoulli's Principle vs. Continuity Equation

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Motivation

- Students show difficulties in solving problems including steady, incompressible pipe flow
- Numerous misconceptions in fluid mechanics have already been reported in hydrostatics [Lov10]
- Fluid dynamics is more complex. It involves difficulties in kinematics and dynamics, it is related to electrical currents, but also shows own characteristic features
- Basic studies of students' thinking concerning this matter are at the beginning [Bro17, Sua17]

Goals

- Analyze students' thinking concerning fluid dynamics
- Identify more general difficulties
- Develop teaching material (tutorials analogue to [McD09]) to improve the conceptual understanding

Investigation

- 304 engineering students of five different programs (first and second year) at the University of Applied Sciences Rosenheim, Germany, 2014/15 and 2017/ 2018
- Various lecturers using various teaching methods (traditional, JiTT/PI, with and w/out corresponding lab)
- Written questionnaire with two multiple choice questions and a subsequent free-response format asking for reasoning (~ 5 min), complete present student group
- Quantitative analysis of correct crossing
- Carefully analysis of the answers, substracting thinking patterns, developping categories and classification of free text answers in these categories.

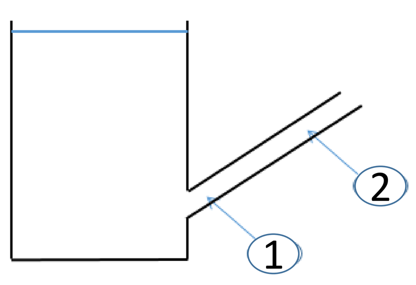
Questionnaire

Q1

Question 1: Water permanently flows out of a wide, water-filled tank through an inclined pipe with constant diameter. The water-level of the tank is kept constant by a refill mechanism. Consider the water flowing in the pipe. Is the velocity of the water in the middle of the pipe at position 2

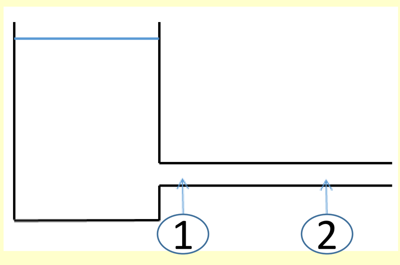
☐ greater than
☐ less than
☐ the same as

the velocity at position 1?

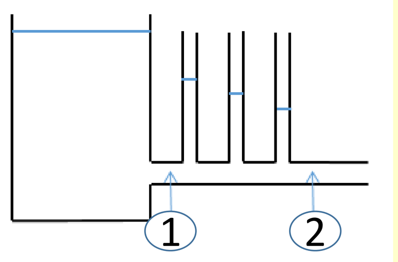


Please explain, why it is like that:

Q2A Question 2A
similar situation
a) same question as in Q1 with friction to be neglected.
b) Is your answer changing, if friction is to be taken into account? Explain.



Q2B Question 2B (alternative)
similar situation
a) same question as in Q1
b) Explain the water-level of the manometer tubes.



Physics background, terminology and conventions

Steady, incompressible pipe flow

- Mass conservation (continuity equation)

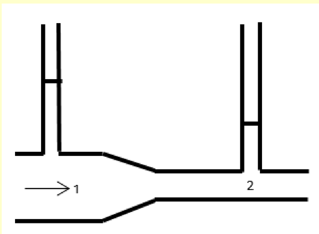
$$\dot{V} = \frac{\Delta V}{\Delta t} = A \cdot \bar{v} = \text{const.}$$

- Energy conservation (Bernoulli's principle) including pressure losses

$$p_1 + \frac{1}{2}\rho \cdot \bar{v}_1^{\,2} + \rho g z_1 = p_2 + \frac{1}{2}\rho \cdot \bar{v}_2^{\,2} + \rho g z_2 + \Delta p_{V12}$$

p_i: static pressure, $\frac{1}{2}\rho \cdot \bar{v}_i^{\,2}$: kinetic energy per volume, $\rho g z$: potential energy per volume, Δp: pressure loss between position 1 and 2

- Venturi nozzle commonly used example for the combination of both principles



Quantitative Results

First-year students:
Increasing level of interaction with the subject from:
group 1: self-study
group 2: traditional lecture
group 3: traditional lecture + lab
group 4: JiTT/PI-lecture + lab (three different courses, three different lectures) leads to an increasing percentage of correct answers:

Gravity: Question 1:
- correct answer checked: 45% (group 1) increasing to 77% (group 4)
- correct reasoning: 20% (gr 1) to 65% (gr 4) correspondingly

Ideal Fluid: Question 2Aa):
- correct answer checked: 70% (gr 1) to 96% (gr 4)
- correct reasoning: 10% (gr 1) to 88% (gr 4)

Friction: Question Q2Ab)
But: even if the answer for ideal fluid is correct, the concept of continuity can be cancelled by friction.
- Correct answer checked and reasoning for Q2Ab) (including also correct reasoning for Q1 and Q2Aa): 5% (gr 1) to 45% (gr 4)

Pressure: Question Q2B
- correct answer checked: 18% (gr 1) to 88% (gr 4)
- correct reasoning: 5% (gr 1) to 40% (gr 4)

Analysis of Student reasoning

1. Failure to use continuity equation
2. Pressure Arguments
   a) Formal arguments using pressure and Bernoulli's equation
   b) Descriptive Arguments using static pressure
3. Analogy to single particle motion
   a) Arguments using gravity or potential energy
   b) Belief of kinetic energy loss due to friction
4. General difficulties
   a) Difficulties with equations containing multiple variables – omitting difficult terms
   b) Understanding the cumulative effect of friction regarding dissipative losses
   c) Recognizing the hierarchy of different principles

Uncertainty in Prioritizing
Q1: [same] "Since the cross section of the pipe stays the same, the velocity stays the same.
[less than] Because of the difference in height, the pressure is changing and hence the velocity.
[same] Cross section stays the same => velocity stays the same." [107]

"Higher pressure means lower velocity"
Q1: [less] By reason of the Bernoulli equation, we know, that at position 2 the dynamic pressure is smaller, because the sum of the pressures remains the same.

Q2B a) [greater] In the water-filled tubes, we see, that the static pressure within [along] the pipe becomes smaller, therefore the dynamic pressure must have *increased (because there is no slope present). [351]*

"Slowing down due to upwards motion"
Q1 [less] „At position 2 the potential energy is higher than at position 1 -> a fraction of the kinetic energy at position 1 will be converted to potential energy at position 2." [349]

Omitting difficult terms
Q1: [same]; [20]

$$p_{stat1} + p_{dyn1} + p_{geo1} = p_{stat2} + p_{dyn2} + p_{geo2}$$
$$p_{geo1} < p_{geo2}; p_{stat1} > p_{stat2} \Rightarrow p_{dyn1} = p_{dyn2}$$

"Lower pressure means lower velocity"
Q1: [less] "Since the pressure decreases, the velocity also decreases." [65]

Q2A a) [less] "The static pressure decreases along the pipe. Therefore the velocity at position 2 is smaller than at position 1." [219]

Q2B a) [less] "As a result of the pressure drop, the velocity of the water decreases in the center of the pipe at position 2." [6]

"Slowing down due to friction"
Q1 [same] "The velocity depends on volume flow and the cross-section of the pipe. Since both does not change, the velocity is the same at 1 and 2."
Q2A a) [same] "In an ideal flow (without friction) there are no losses. Moreover, the cross-section and the volume flow are constant."
Q2A b) "Yes [i.e. the answer changes], position 2 has a smaller velocity, since the velocity in the pipe decreases due to friction." [276]

Hierarchy of principles not recognized
Q1: [same], „velocity is constant, because the volume flow and the diameter of the pipe is constant 2.
Q2Ba) [greater] „As the pressure at position 2 has dropped, the velocity must have risen"[57]

Cumulative effect of friction not seen
"Q2A b) „If the friction is the same at 1 and 2, then the velocity is the same as well. If it was to be different, it would change …."[68]

Summary

Minimum percentage of first-year students **who failed** to apply **the concept of mass conservation**:

- 35% when the situation involved gravity (Q1),
- 55% when the situation involved friction (Q2A), and
- 60% when the situation involved a pressure drop (Q2B).

- Lack of understanding the hierarchy of the concepts
- Frequently, the role of pressure is not clear
- Experiences from single particle motion are transferred to fluid mechanics (gravity, friction)
- More general difficulties can be found

Outlook
- Test tutorial for continuity equation
- Develop tutorial for Bernoulli equation

References:

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